$$16h^8d^4(d-1)^2-16h^7d^3(4d^2-17d+5)-4h^6d^2(4d^3-31d^2+219d-37)$$

$$+4h^5d(10d^3-80d^2+337d-30)+h^4(4d^4-40d^3+682d^2-997d+36)$$

$$-h^3(4d^3-92d^2+679d-285)+h^2(2d^2-147d+241)-h(9d-71)+7=0.$$

When $d=0$, corresponding to the cardioid, only one of the four roots is available, two of the others being excluded since $b\neq 0$ and the fourth one because $b\neq\infty$, already included in the second special case.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

245. Proposed by S. I. JONES, A. B., Gunter Bible College, Gunter, Texas.

The shell of a hollow iron ball is 4 inches thick, and contains $\frac{1}{5}$ of the number of cubic inches in the whole ball. Find the diameter of the ball.

I. Solution by S. A. COREY, Hiteman, Iowa.

Let r be the radius of the ball; $(r-4)$ will then be the radius of the hollow sphere enclosed by the shell. As the volumes of spheres are proportional to the cubes of their radii, the conditions of the problem require that

$r^3-(r-4)^3=\frac{1}{5}r^3$, or $\frac{4}{5}r^3=(r-4)^3$, whence, $r=\dfrac{4}{1-\sqrt[3]{\frac{4}{5}}}=55.79$ inches, nearly.

II. Solution by M. R. BECK, Cleveland High School, Ohio.

Let r be the radius of the ball, then $\frac{4}{3}\pi r^3=\frac{4}{15}\pi r^3+\frac{4}{3}\pi(r-4)^3$..........(1).
From (1) we have $r^3-60r^2+240r-320=0$.........(2).
Substitute $r=x+\dfrac{320}{x}+20$ in (2), $x^3+\dfrac{32,768,000}{x^3}-11520=0$..........(3).
Solving the quadratic (3), $x=\sqrt[3]{(6400)}$ or $\sqrt[3]{(5120)}$.
Either root makes $r=55.8016$, and the diameter is 111.6032 inches.

Also solved by P. S. Berg, G. W. Greenwood, A. H. Holmes, L. E. Newcomb, D. B. Northrup, J. Scheffer, J. E. Sanders, and G. B. M. Zerr.

AVERAGE AND PROBABILITY.

169. Proposed by HENRY HEATON, Atlantic, Iowa.

*What is the average length of all straight lines that can be drawn within a given square in every possible direction and every possible length from every point of the square; if all the lines are equally distributed about the starting point and equally distributed as to length.

*The problem as restated above is somewhat different from the one solved in our columns last month. As the above conveys the original meaning of the Proposer it is published as a third solution.